PIT Project Research Report: TM3

Study of thermal expansion and bowing in a restrained beam

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March 2000

Contents

1 Abstract

This report describes a study carried out on a simple ABAQUS beam model to investigate and understand the behaviour of the beam when the deflection response is governed primarily by restrained thermal expansion and thermal bowing. The study was developed to calibrate and test the results of a theoretical analysis on a simple beam model described in detail in report TM2.

2 Introduction

In all the Cardington Frame tests conducted by the Building Research Establishment (BRE) and British Steel and in accidental compartment fires in redundant, multi-storey, steel framed composite buildings, very large deflections can be observed in the beam and slab elements in the region of the fire.

The current understanding of causes of large deflections in beams subjected to thermal loading are

- Thermal expansion is the primary cause.
- Thermal gradients play a supporting role.
- Thermal expansion can only produce large deflections if the ends are restrained from expanding.
- Thermal gradients can only produce deflections if the ends are free to rotate (by imposing curvature).
- Material degradation and loading are secondary influences on deflections

3 The Models

Figure 1 shows the ABAQUS beam models studied. The model characteristics are as follows,

- Length=l
- slenderness ratio (fixed ends), $l/r = 35$
- slenderness ratio (pinned ends), $l/r = 70$
- $Area, A = 5160mm^2$
- Thermal expansion coefficient, $\alpha = 8 \times 10^{-6}mm/^oC$

- Uniformly Distributed Load, $UDL = 0.1N/mm$

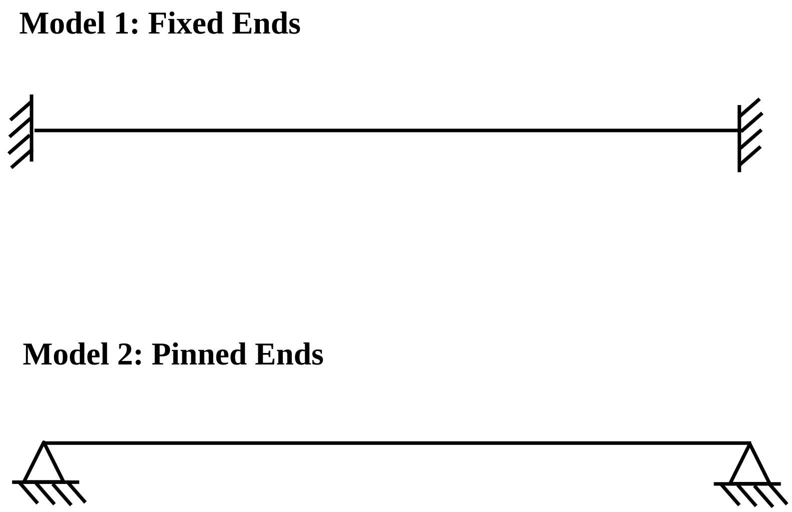

Figure 1: The models studied

The heating regime applied to the model included,

- A mean temperature rise (uniform over the length)= ΔT
- An Effective thermal gradient through the depth= $T,_y = \frac{T_2 - T_1}{d}$

and is described in figure 2.

The temperature increase, ΔT and thermal gradient, T, y are applied to the simple beam model at a constant rate from zero to their maximum values.

4 Structural response

The structural response of the model as a result of the heating regime is briefly described below.

For a uniform temperature increase with no gradient,

- the thermal expansion strain= $\epsilon_T = \alpha \Delta T$
- therefore the thermal expansion $= \epsilon_T l$

and for an effective thermal gradient through the depth of the model with no mean temperature rise

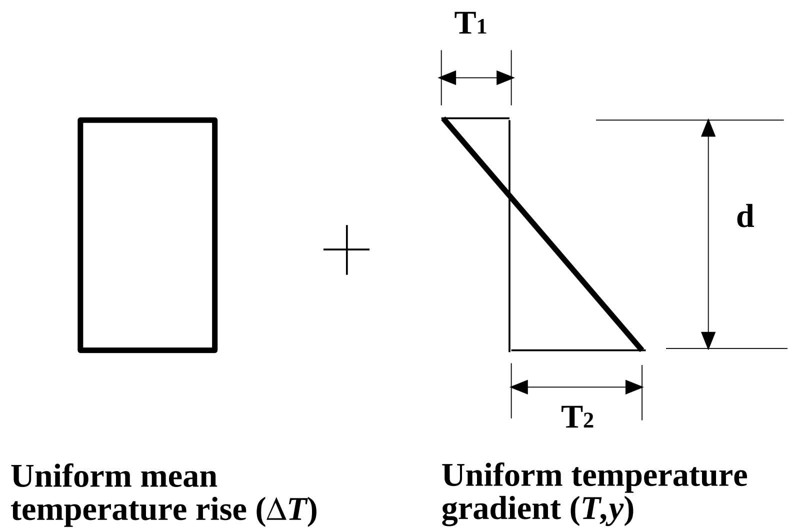

Figure 2: The heating regime applied to the models

- thermal bowing induced curvature $= \phi = \alpha T_{,y}$
- thermal bowing induced contraction strain$= \epsilon_{\phi} = 1 - \frac{\sin\frac{l\phi}{2}}{\frac{l\phi}{2}}$
- thermal bowing induced contraction$= \epsilon_{\phi} l$

When combinations of ϵ_T and ϵ_{ϕ} are applied the response of the model is interpreted based upon the fundamental relationships for thermal stress as described by equations 1, 2 and 3.

$$\epsilon_{\text{total}} = \epsilon_{\text{thermal}} + \epsilon_{\text{mechanical}} \tag{1}$$

with

$$\epsilon_{\text{mechanical}} \rightarrow \sigma \tag{2}$$

and

$$\epsilon_{\text{total}} \rightarrow \delta \tag{3}$$

Equations 1, 2 and 3 state that the total strains govern the deformed shape of the structure δ, but the stress state in the structure σ (elastic or plastic) depends only on the mechanical strains. Mechanical strains arise as a result of loading or restrained thermal expansion.

Where the thermal strains are free to develop in an unrestricted manner and there are no external loads, axial expansion or thermal bowing result as described by equations 4 and 5.

Boundary conditions	Mean temperature rise (ΔT)	Gradient ($T_{,y}$)
Fixed ends	400^oC	
Fixed ends	100^oC	$1^oC/mm$
Fixed ends	200^oC	$1^oC/mm$
Fixed ends	400^oC	$1^oC/mm$
Fixed ends	100^oC	$5^oC/mm$
Fixed ends	200^oC	$5^oC/mm$
Fixed ends	400^oC	$5^oC/mm$

Table 1: Conditions in each run on Model 1

$$\epsilon_{\text{total}} = \epsilon_{\text{thermal}} \quad (4)$$

and

$$\epsilon_{\text{total}} \rightarrow \delta \quad (5)$$

Where thermal expansion strains are fully restrained without external loads, thermal stresses and plastification result from equations 6 and 7.

$$0 = \epsilon_{\text{thermal}} + \epsilon_{\text{mechanical}} \quad (6)$$

with

$$\epsilon_{\text{mechanical}} \rightarrow \sigma \quad (7)$$

For a full explanation of the theory used to describe the response of structures to fire see reports TM1 and TM2.

5 Studies on Model 1 (Fixed ends)

Figure 3 shows the fully restrained model (axially and rotationally restrained) subjected to a mean temperature rise and a thermal gradient. The combination of mean temperature rise, ΔT, and axial restraint causes compressive axial forces to develop in the beam as it tries to expand. Whilst the thermal gradient induces a uniform curvature $\phi = \alpha T_{,y}$, equal and opposite to the curvature induced by the support moments therefore the beam remains straight with a constant moment, $M = EI\phi$, along its length. The analyses carried out on the fully fixed beam are listed in table 1.

The results of these analyses were as expected. Plots of deflection, axial force and moment against temperature are given in figures 4, 5 and 6 for two scenarios, a uniform mean temperature rise of 400^oC and a combination of mean

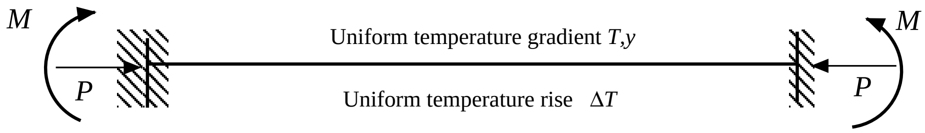

Figure 3: Fixed beam theory

temperature and gradient. In both instances the deflections are almost zero, while the axial force due to thermal expansion is in compression and the moments are zero for pure thermal expansion and increasing uniformly in hogging with the increase in gradient.

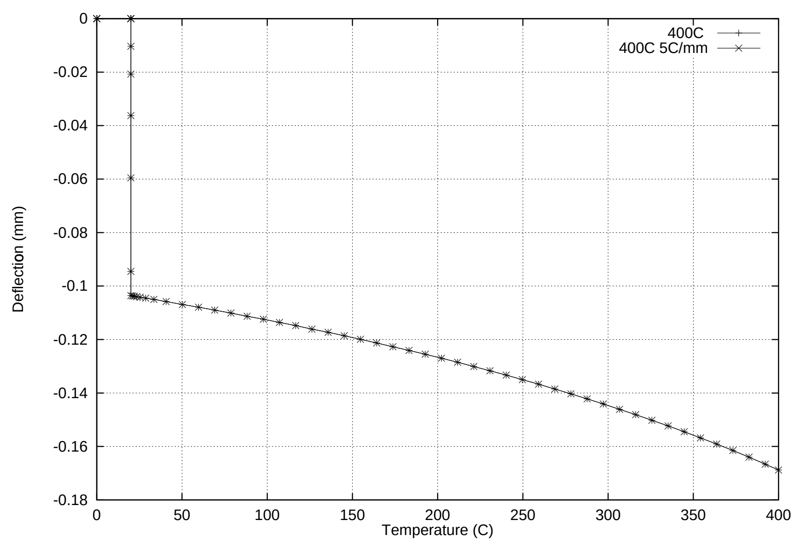

Figure 4: Deflections of the fully fixed beam in response to thermal expansion and thermal bowing

6 Studies on Model 2 (Pinned ends)

Table 2 lists the initial studies on the pinned beam (laterally restrained but free to rotate at its ends). In the first three analyses the effect of thermal expansion and thermal bowing were studied alone.

6.1 Restrained thermal expansion

If there is no thermal gradient in the beam but a uniform temperature rise is applied. The only response will be restrained thermal expansion. There are

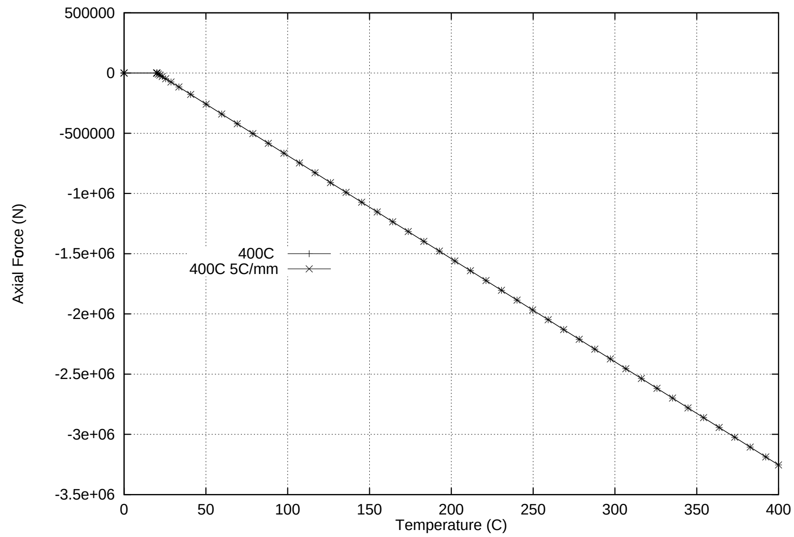

Figure 5: Axial forces in the fully fixed beam in response to thermal expansion and thermal bowing

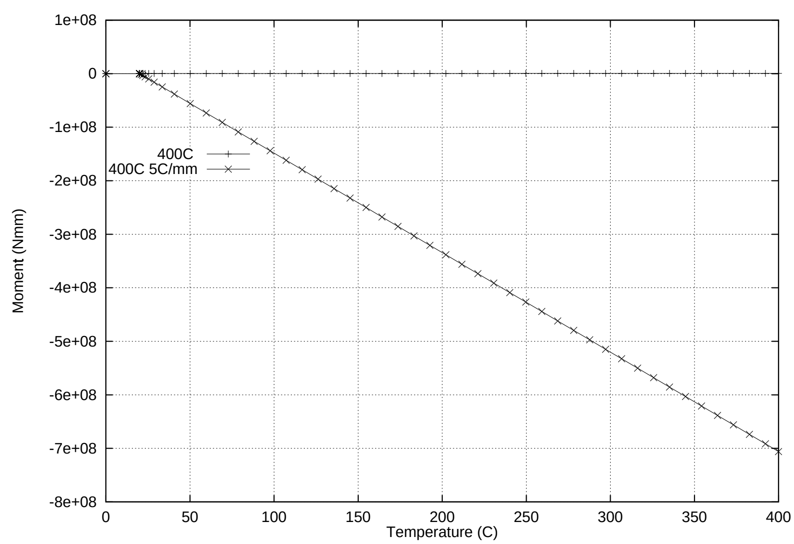

Figure 6: Moments in the fully fixed beam in response to thermal expansion and thermal bowing

Boundary conditions	Mean temperature rise (ΔT)	Gradient ($T_{,y}$)
Pinned ends	400^oC	
Pinned ends		$1^oC/mm$
Pinned ends		$5^oC/mm$

Table 2: Conditions in each run on Model 2

two distinct stages in the response of the beam. A pre-buckling phase and a post-buckling phase. These are illustrated in the first two images in figure 7.

In the **pre-buckling** phase the thermal expansion is absorbed in elastic (or plastic) strains and very little displacement is produced. The thermal and mechanical strains are equal. High stresses are generated until the beam reaches its limit and buckles.

Once buckling occurs all further thermal strains produce large deflections.

Figure 8 shows the result from the ABAQUS numerical model. The four frames in figure 8 show the model and three plots, mid-span deflection, axial force and moment plotted against mean temperature. The double curvature of the pre-buckling / post buckling phase is clearly evident in the deflected shape and the axial force is in pure compression.

6.2 Thermal Bowing

In this instance if there is no mean temperature rise applied to the beam just a thermal gradient the axial forces are caused by tensile 'contraction' strains as a result of the thermal curvature imposed on the beam trying to pull the beam ends in. This is illustrated in the third image of figure 7.
Figures 9 and 10 show the numerical results for $T_{,y} = 1^oC/mm$ and $T_{,y} = 5^oC/mm$ (the temperature on the x-axis is negative because ABAQUS plots the deflections etc against the top flange temperature or coldest surface). In both cases axial force is in tension and moment in hogging bending as expected. The deflection response at mid-span for $T_{,y} = 1^oC/mm$ is linear whereas the response at $T_{,y} = 5^oC/mm$ is very non-linear.

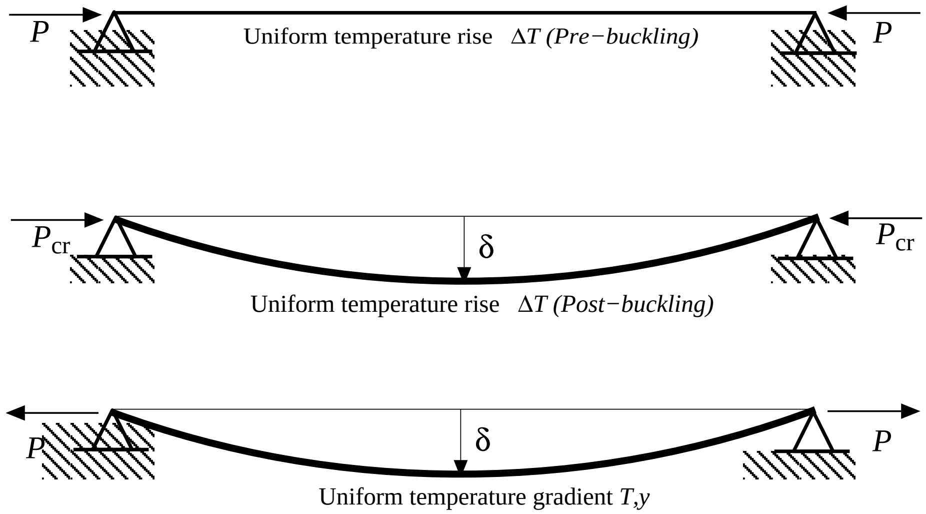

Figure 7: Pinned beam theory

6.3 Combined thermal expansion and thermal bowing

In the numerical study discussed here the temperature increase ΔT and the thermal gradients $T_{,y}$ are applied to the model at a constant rate from zero to their maximum value. The strains induced in the beam by the two effects are in opposition to each other. This situation leads to an effective strain as described by equation 8.

$$\epsilon_{\text{eff}} = \epsilon_T - \epsilon_\phi \tag{8}$$

The final set of analyses carried out on the pinned end model are listed in table 3.

Depending on the magnitude of the mean temperature rise in relation to the magnitude of the thermal gradient there are a range of responses in terms of deflections, axial forces and moments as a result of thermal expansion and thermal bowing interaction. The thermal expansion produced is partly used up in generating mechanical strains and partly in generating deflections. This is governed by equation 8. ϵ_{eff} is the component that generates stresses to push the beam towards buckling. The ϵ_ϕ component takes part of the expansion and produces deflections by imposing curvature with the available excess length.

Figures 11, 12 and 13 show the results from a selection of the analyses listed in table 3. In terms of deflection for a pure thermal expansion with no thermal gradient applied to the model the response is the double curvature of the

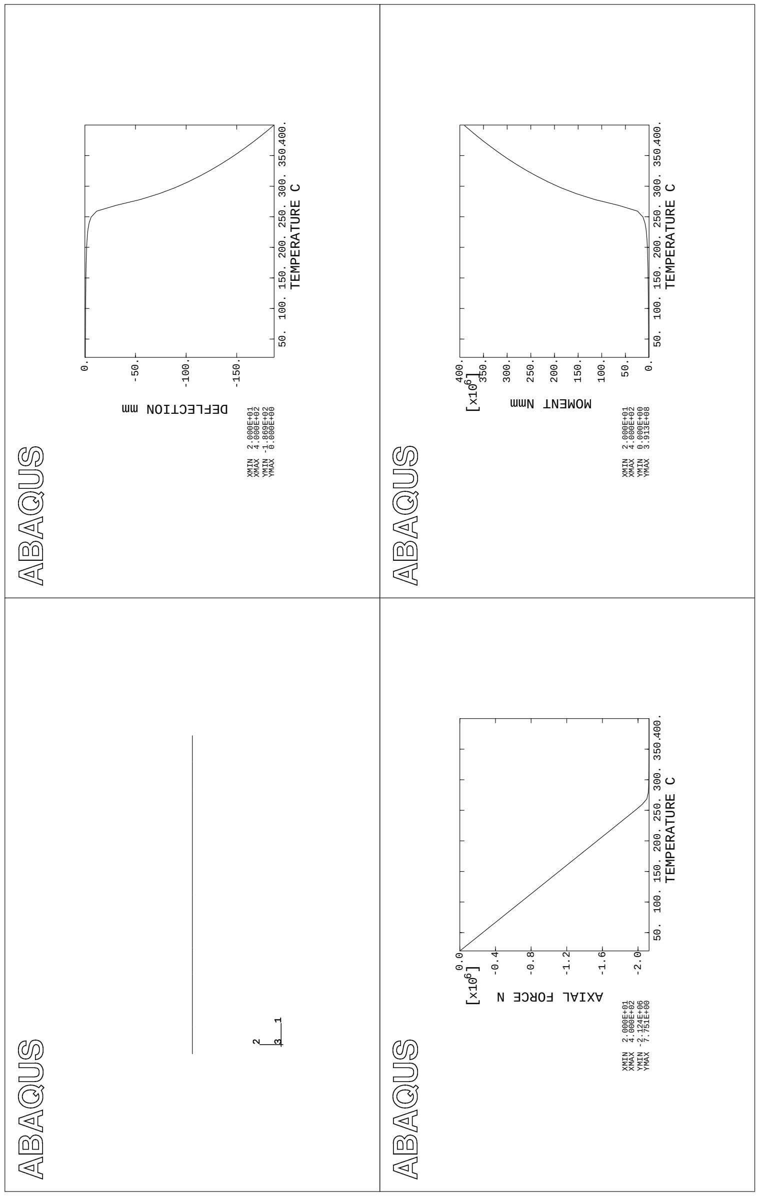

Figure 8: Numerical Model:Thermal Expansion Only

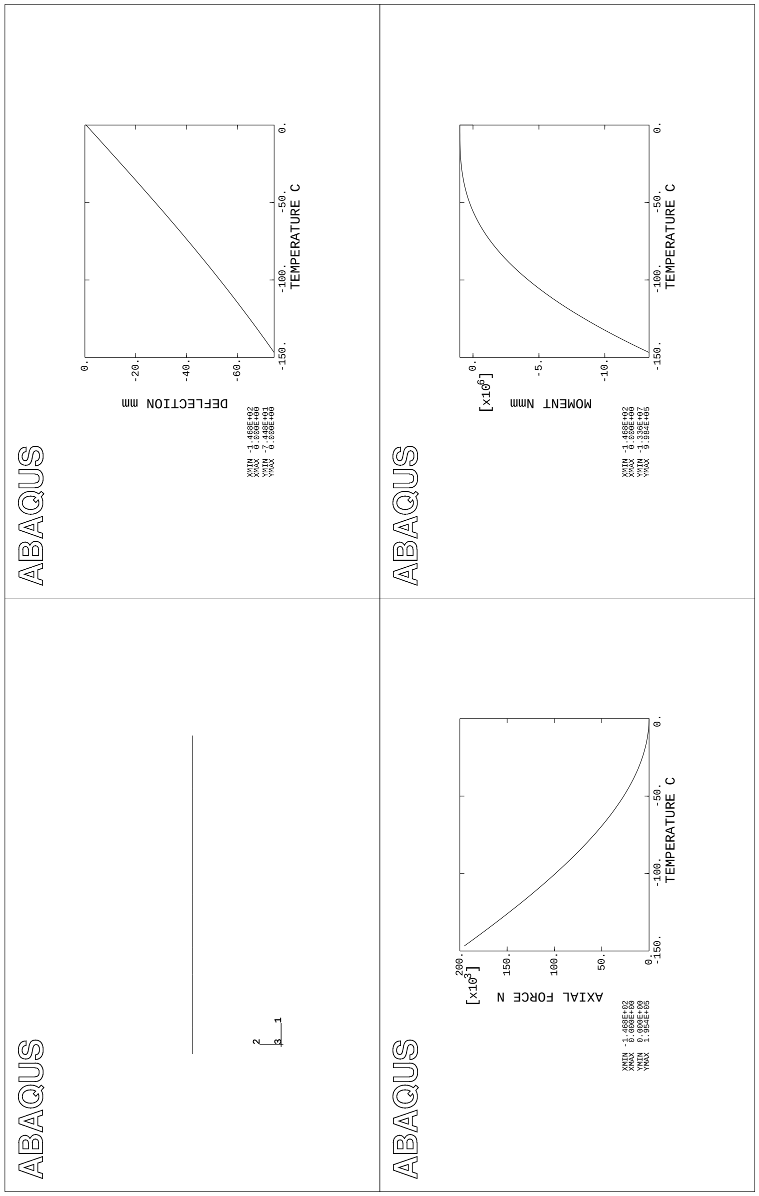

Figure 9: Numerical Model:Deflection due to Thermal Bowing Only, $T_{,y} = 1^oC/mm$

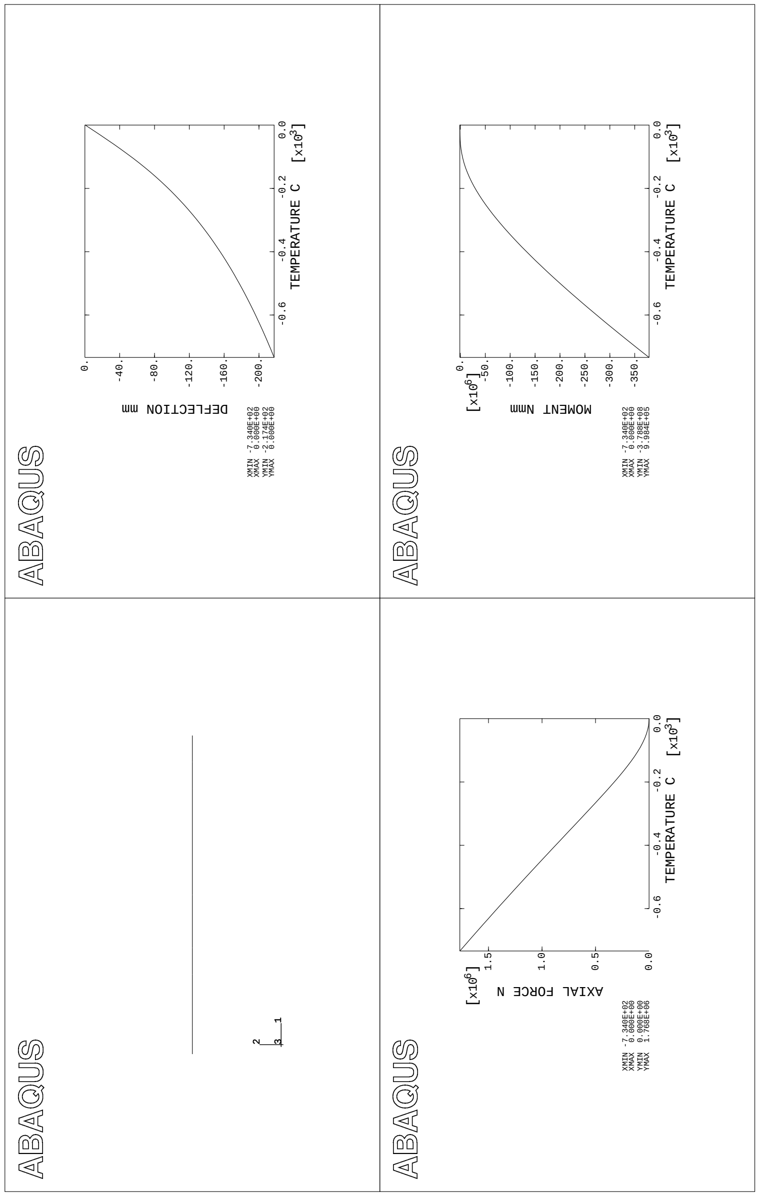

Figure 10: Numerical Model:Deflection due to Thermal Bowing Only, $T_{,y} = 5^oC/mm$

Boundary conditions	Mean temperature rise (ΔT)	Gradient ($T_{,y}$)
Pinned ends	100^oC	$1^oC/mm$
Pinned ends	200^oC	$1^oC/mm$
Pinned ends	400^oC	$1^oC/mm$
Pinned ends	100^oC	$5^oC/mm$
Pinned ends	200^oC	$5^oC/mm$
Pinned ends	400^oC	$3^oC/mm$
Pinned ends	400^oC	$5^oC/mm$
Pinned ends	400^oC	$10^oC/mm$

Table 3: Conditions in each run on Model 2

pre-buckled/post-buckled form. As a thermal gradient is incorporated into the analyses the deflections increase and the overall shape becomes smoother. The axial forces (figure 12) gradually decrease in compression and move into high tension as the thermal gradient applied increases. Finally in terms of the moments as the gradients increase the beam moves into hogging bending.

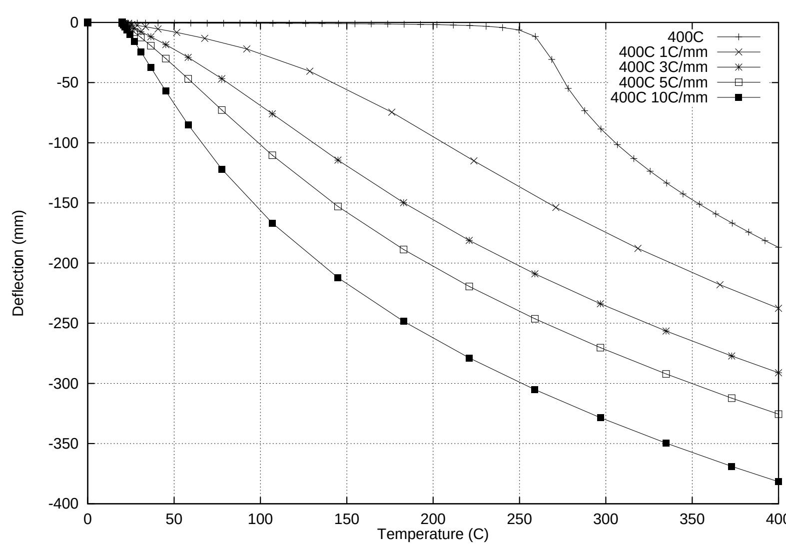

Figure 11: Deflections of the pinned beam at mid-span in response to thermal expansion and thermal bowing

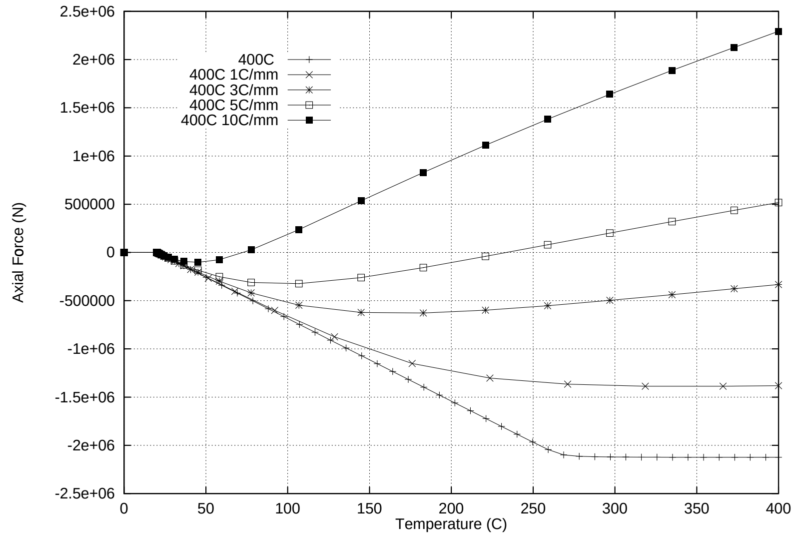

Figure 12: Axial forces in the pinned beam in response to thermal expansion and thermal bowing

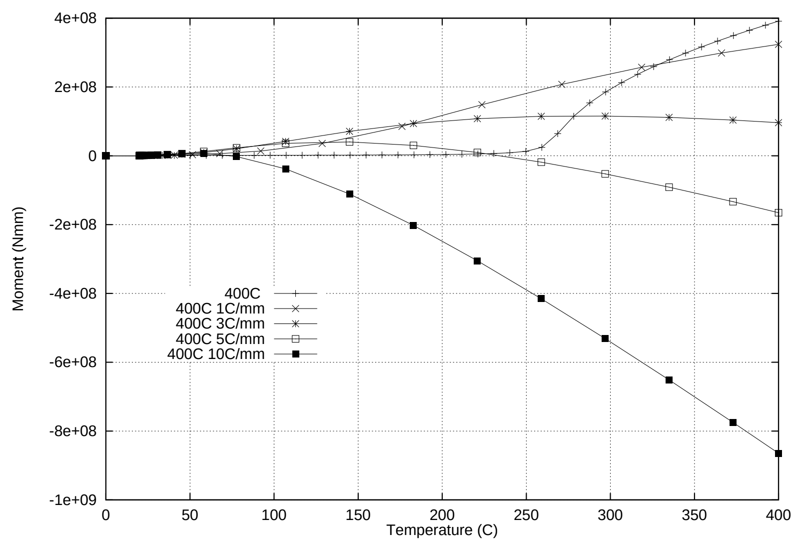

Figure 13: Moments in the pinned beam in response to thermal expansion and thermal bowing

7 Conclusions

- The results from the simple numerical model highlight the key responses to restrained thermal expansion and thermal bowing
- Fully restrained beams respond to thermal expansion with very small deflections but high compressive axial forces and to thermal bowing with no deflections but a uniform moment along the beams length.
- A beam which is free to rotate at its ends will buckle under restrained thermal expansion at a critical temperature and large deflections will develop.
- Thermal expansion and thermal bowing interaction produce a large number of responses in terms of deflections, axial forces and moments in a laterally fixed but rotationally free beam.